Texas A&M University Mathematics Practice Placement Exam - Solutions

1. Find the x and y intercepts for the function $f(x) = x^3 - 9x$.

   Solution: To find x intercepts, solve $f(x) = 0$:

   $x^3 - 9x = 0 \Rightarrow x = 0, \pm 3$. To find the y intercept, substitute $x = 0$: $f(0) = 0$. Hence the y intercept is $y = 0$.

2. Find the domain of:

   (a) $f(x) = \sqrt{-x^2 - 4x + 5}$

   Solution: We must have $-x^2 - 4x + 5 \geq 0$ in order for $\sqrt{-x^2 - 4x + 5}$ to be defined. Thus $(-x + 1)(x + 5) \geq 0$, giving us $-5 \leq x \leq 1$.

   (b) $g(t) = \ln(4t - 3)$

   Solution: We must have $4t - 3 > 0$ in order for $\ln(4t - 3)$ to be defined. This yields $t > \frac{3}{4}$.

   (c) $h(x) = \dfrac{1}{x^3 + 3x^2 - x - 3}$

   Solution: We must have $x^3 + 3x^2 - x - 3 \neq 0$ in order for $\dfrac{1}{x^3 + 3x^2 - x - 3}$ to be defined. $x^3 + 3x^2 - x - 3 \neq 0 \Rightarrow$ $(x^2 - 1)(x + 3) \neq 0$, yielding $x \neq \pm 1$ and $x \neq -3$. Thus the domain is all real numbers except $x = \pm 1$ and $x = -3$.

3. Simplify the expression. Write your answer using positive rational exponents. $\left(\dfrac{2}{\sqrt{x^5}}\right)\left(\sqrt[3]{4x}\right)$

   Solution: $\left(\dfrac{2}{\sqrt{x^5}}\right)\left(\sqrt[3]{4x}\right) = (2x^{-5/2})(2^{2/3}x^{1/3}) = \dfrac{2^{5/3}}{x^{13/6}}$

4. If we begin with the graph of $f(x) = \sqrt{x}$, shift 4 units to the right, shrink vertically by a factor of $\frac{1}{2}$, and shift upward 10 units, write an equation for the transformed graph.

   Solution: The transformed graph is $g(x) = \frac{1}{2}\sqrt{x - 4} + 10$.

5. Solve for x: $\log(x+2)+\log(x-1)=1$.

   Solution: Use properties of logarithms:

   $\log(x+2)+\log(x-1)=1 \Rightarrow \log(x+2)(x-1)=1 \Rightarrow$

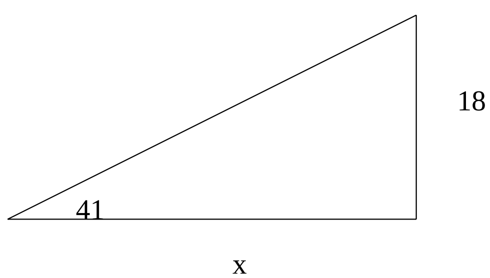
   $(x+2)(x-1)=10$. This yields $x=-4$ and $x=3$. Since $x=-4$ is not in the domain, $x=3$ is the only solution.

6. Factor completely: $3x^2(4x^2+1)^8+64x^4(4x^2+1)^7$.

   Solution: $3x^2(4x^2+1)^8+64x^4(4x^2+1)^7=x^2(4x^2+1)^7(76x^2+3)$.

7. How far from the base of an 18 foot tall pole must a person be standing if the angle of elevation from the ground to the pole is 41°?

   Solution: Refer to the figure below. We know that $\tan(41°)=\dfrac{18}{x}$, thus $x=\dfrac{18}{\tan(41°)}$, or $x=18\cot(41°)$.



8. Find $f\circ g$ if $f(x)=\dfrac{x}{x+1}$ and $g(x)=\dfrac{2}{x}$. Simplify.

   Solution: $f\circ g=\dfrac{\frac{2}{x}}{\frac{2}{x}+1}=\dfrac{2}{2+x}$.

9. Perform the indicated operation and simplify: $\dfrac{8}{x+1}-\left(\dfrac{y}{z+2}\div\dfrac{y-4}{w}\right)$

   Solution: After getting a common denominator of $(x+1)(z+2)(y-4)$, we find that

   $$\frac{8}{x+1}-\left(\frac{y}{z+2}\div\frac{y-4}{w}\right)=\frac{8zy-32z+16y-64-ywx-yw}{(x+1)(z+2)(y-4)}.$$

10. Solve for x: $e^{2x}-2e^x-3=0$.

    Solution: $e^{2x}-2e^x-3=(e^x)^2-2x^x-3=(e^x-3)(e^x+1)$. Solving $(e^x-3)(e^x+1)=0$ yields $x=\ln 3$ as the only solution.

11. Find the equation of the line passing through the point $(5, 1)$ with slope 7. Next, find y when $x = -4$.

    Solution: The equation of the line is $y - 1 = 7(x - 5)$. Thus if $x = -4$, $y = -62$.

12. If $f(x) = \sqrt{x+4}$, find and simplify $\dfrac{f(2+h) - f(2)}{h}$.

    $\dfrac{f(2+h) - f(2)}{h} = \dfrac{\sqrt{6+h} - \sqrt{6}}{h}$. Multiply the numerator and denominator by $\sqrt{6+h} + \sqrt{6}$ yields a simplified answer of $\dfrac{1}{\sqrt{6+h} + \sqrt{6}}$.

13. Simplify $\dfrac{(x^2y^4)^5(x^3y)^{-3}}{xy}$.

    Solution: $\dfrac{(x^2y^4)^5(x^3y)^{-3}}{xy} = \dfrac{x^{10}y^{20}x^{-9}y^{-3}}{xy} = y^{16}$.

14. Simplify $\sqrt[3]{a^3b}\sqrt[3]{64a^4b^2}$.

    Solution: $\sqrt[3]{a^3b}\sqrt[3]{64a^4b^2} = \sqrt[3]{64a^7b^3} = 4a^2b\sqrt[3]{a}$.

15. Perform the operations and simplify.

    $\dfrac{x^2}{x^2 - x - 2} - \dfrac{4}{x^2 + x - 6} + \dfrac{x}{x^2 + 4x + 3}$.

    Solution: Factor the denominators:

    $\dfrac{x^2}{(x-2)(x+1)} - \dfrac{4}{(x+3)(x-2)} + \dfrac{x}{(x+3)(x+1)}$. After getting a common denominator of $(x+1)(x-2)(x+3)$, we obtain $\dfrac{x^3 + 4x^2 - 6x - 4}{(x+1)(x-2)(x+3)}$.

16. Find all zero's and vertical asymptotes for $f(x) = \dfrac{3x^2 - 14x - 5}{4x^2 - 17x - 15}$

    Solution: Factor and simplify: $f(x) = \dfrac{3x+1}{4x+3}$. This yields a zero of $x = -\dfrac{1}{3}$ and a vertical asymptote of $x = -\dfrac{3}{4}$.

17. If θ is in quadrant II and $\sin\theta = \frac{1}{7}$, what is $\cos\theta$?

Solution: Using the identity $\cos^2\theta + \sin^2\theta = 1$, we find that $\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{1}{49}$. This gives us $\cos\theta = \pm\frac{\sqrt{48}}{7}$. Since θ is in quadrant II, $\cos\theta < 0$, hence $\cos\theta = -\frac{4\sqrt{3}}{7}$.

18. Use properties of logarithms to expand the expression $\ln\left(\frac{\sqrt{x}y^5}{(z+1)^4}\right)$.

Solution: We know $\ln ab = \ln a + \ln b$; $\ln\frac{a}{b} = \ln a - \ln b$ and $\ln a^b = b\ln a$. Using these properties, we obtain $\ln\left(\frac{\sqrt{x}y^5}{(z+1)^4}\right) = \frac{1}{2}\ln x + 5\ln y - 4\ln(z+1)$.

19. Evaluate $\sec\frac{2\pi}{3} - \tan\frac{\pi}{6}$.

Solution: $\sec\frac{2\pi}{3} - \tan\frac{\pi}{6} = -2 - \frac{1}{\sqrt{3}}$.

20. If we begin with a rectangle with length 5 inches and width 4 inches, then increase the length by 8%, what is the change in area?

Solution: We begin with a rectangle of area 20 square inches. If we increase the length by 8%, then our new length is 5.4 inches. This gives a new area of 21.6 square inches. This gives a total change in area of 1.6 square inches.

21. Evaluate $f(2) - f(-3)$ If

$$f(x) = \begin{cases} x^3 + 1 & , \quad \text{if } x > 1 \\ 2x^2 - 3 & , \quad \text{if } x \leq 1 \end{cases}$$

Solution: $f(2) = 9$ and $f(-3) = 15$. Thus $f(2) - f(-3) = -6$.

22. Simplify the expression $\frac{\cos^2\theta}{1+\sin\theta}$.

Solution: Use the identity $\cos^2\theta = 1 - \sin^2\theta$. Then after using the difference of two squares, we obtain a simplified answer of $1 - \sin\theta$.

23. Evaluate $\log_4 \frac{1}{\sqrt[3]{16}}$.

Solution: Using the fact that $\log_b b^x = x$, we find that $\log_4 \frac{1}{\sqrt[3]{16}} = -\frac{2}{3}$.

24. Simplify $\frac{\frac{1}{a} - b}{\frac{1}{b^3} + a}$.

Solution: Multiply the numerator and denominator by ab^3. This gives us $\frac{\frac{1}{a} - b}{\frac{1}{b^3} + a} = \frac{b^3 - ab^4}{a + a^2b^3}$.

25. A bacteria culture contains 1200 bacteria and doubles every day. How many hours will it take the culture to reach 10000 bacteria?

Solution: Use the exponential growth model: $y(t) = y_0e^{kt}$, where $y(t)$ is the size of the population at time t and y_0 is the initial size of the population. We know that $y_0 = 1200$, hence $y(t) = 1200e^{kt}$. Using the fact that the population doubles every day, we find that $k = \ln 2$. Now we will solve $y(t) = 10000$ for t. This gives $t = \frac{\ln(25/3)}{\ln 2}$ days, hence $t = 24(\frac{\ln(25/3)}{\ln 2})$ hours.