Problem 1. Rationalize the denominator of the fraction $\frac{14}{3+\sqrt{2}}$.

Solution: Multiply the numerator and denominator by the conjugate of $3+\sqrt{2}$. After simplifying, this gives $6-2\sqrt{2}$. □

Problem 2. Given $g(x) = 5x^2 - 2x + 3$ and $h(x) = \sqrt{9x+7}$, find the value of $(g \circ h)(2)$.

Solution: Since $h(2) = 5$, this gives $g\big(h(2)\big) = g(5) = 118$. □

Problem 3. Find the sum below, and write your answer in simplified form.

$$\frac{x+2a-3}{x+a} + \frac{x+6}{2x}.$$

Solution: Re-write both fractions with common denominator $2x(x+a)$ before addition. After simplifying, this gives

$$\frac{3x^2+5ax+6a}{2x(x+a)}.$$ □

Problem 4. Factor and reduce to simplest form the expression $\frac{6x^2+11xy-10y^2}{3x^2+10xy-8y^2}$.

Solution: Factoring the numerator and denominator gives

$$\frac{(3x-2y)(2x+5y)}{(3x-2y)(x+4y)} = \frac{2x+5y}{x+4y}.$$ □

Problem 5. Simplify completely the expression $\frac{\left(x^{-4}y^{2/5}\right)^{-3/4}}{x^{2/3}y^{-5/6}}$.

Solution: Using properties of exponents, we get $\frac{x^3y^{-3/10}}{x^{2/3}y^{-5/6}} = x^{7/3}y^{8/15}$. □

Problem 6. Perform the indicated operation and simplify your answer:

$$\left(3ab\sqrt[5]{16a^4b^2}\right)\left(8b^7\sqrt[5]{2ab^3}\right).$$

Solution: Simplifying the radicals gives $\left(3ab2^{4/5}a^{4/5}b^{2/5}\right)\left(8b^7 2^{1/5}a^{1/5}b^{3/5}\right) = 48a^2b^9$. □

Problem 7. Find the sum of the solutions of the equation: $5(x-7)^2 - 13(x-7) - 6 = 0$.

Solution: Let $y = x-7$. Solving the resulting quadratic equation and substituting back, we get the sum of the roots is $83/5$. □

Problem 8. A radioactive element decays according to the model $y = 80e^{-0.21t}$, where t is in hours. Use natural logarithms to find the half-life of the element.

Solution: Half-life is the time at which half of the original amount remains. Since the initial amount of this function is 80 units, we must solve for t in the equation $40 = 80e^{-0.21t}$.

This gives $t = -\ln(0.5)/0.21$. □

Problem 9. A radioactive element decays according to the model $y = 80e^{-0.21t}$, where t is in hours. Use natural logarithms to find the half-life of the element.

Solution: Half-life is the time at which half of the original amount remains. Since the initial amount for this function is 80 units, we must solve the equation $40 = 80e^{-0.21t}$, which gives $t = -\ln(0.5)/0.21$. □

Problem 10. Find the point (x, y) which satisfies both equations below. What is the value of $x + y$?

$$\begin{cases} -3 = 3x - 5y \\ 12 = -2x + 4y \end{cases}$$

Solution: Solving the system gives $x = 24$, $y = 15$. The sum is therefore $x + y = 39$. □

Problem 11. Two investments are made totaling \$10,000. In one year these investments yield \$650 in simple interest. Part of the \$10,000 is invested at $5\frac{1}{2}\%$, and the rest at $6\frac{3}{4}\%$. How much more money is invested at $6\frac{3}{4}\%$?

Solution: Let x be the amount invested at $5\frac{1}{2}\%$ and y be the amount invested at $6\frac{3}{4}\%$. We want to compute the difference $y - x$. The values of x and y can be obtained after solving the following system

$$\begin{cases} 10000 = x + y \\ 650 = 0.055x + 0.0675y. \end{cases}$$

This gives $x = 2000$, $y = 8000$, for a difference of \$6,000. □

Problem 12. Let $y = f(x)$ be a linear function that satisfies $f(3) = 5$ and $f(6) = 9$. Find the slope of the line.

Solution: Since the points $(3, 5)$ and $(6, 9)$ are on the line, the slope is

$$\frac{\Delta y}{\Delta x} = \frac{9-5}{6-3} = \frac{4}{3}.$$ □

Problem 13. Write a slope-intercept form of the equation of a line with slope 5 that goes through the point $(-2, 3)$.

Solution: We seek an equation of the form $y = mx + b$, where $m = 5$ is the slope. Since the point $(-2, 3)$ is on the line, we solve for b in the equation $3 = 5(-2) + b$. It is $b = 13$, so the equation we seek is $y = 5x + 13$. □

Problem 14. Find the vertex of the parabola $f(x) = 7px^2 + 28px + 11p$.

Solution: The formula to compute the vertex of a parabola gives $x = -2$. The coordinates of this point are thus $\big(-2, f(-2)\big) = (-2, -17p)$. □

Problem 15. Let $f(x) = \sqrt{x}$. Find the expression of a function $g(x)$ whose graph is that of f shifted to the left 2 units, reflected about the x axis, and then shifted up by 7 units.

Solution:

$$\sqrt{x} \to \sqrt{x+2} \to -\sqrt{x+2} \to -\sqrt{x+2}+7$$

Applying these transformations in order gives $g(x) = 7 - \sqrt{x+2}$. □

Problem 16. Which one of the following is an incorrect algebraic statement?

$$7(x+1)^3 \neq (7x+7)^3 \qquad \frac{1}{3x^8} = \frac{1}{3}x^{-8} \qquad (x+8)^2 = x^2 + 64$$

Solution: The third statement is false. The other two statements are true. □

Problem 17. Find the domain of the function

$$f(x) = \frac{\sqrt{x+2}}{x-5}.$$

Solution: The expression under the square root sign must not be negative. The denominator may not be equal to zero. Using interval notation, we get $[-2, 5) \cup (5, \infty)$. □

Problem 18. Find $f(0)$ for the piecewise function given by

$$f(x) = \begin{cases} x+8 & \text{for } x \geq 3 \\ x-4 & \text{for } x < 3 \end{cases}$$

Solution: $f(0) = 0 - 4 = -4$, since $x = 0$ gets evaluated only by the second piece (since $0 < 3$). □

Problem 19. Find the x–intercept(s) of the following function, if any exist.

$$f(x) = \frac{6x^2 - 7x - 5}{4x^2 - 12x - 7}.$$

Solution: Factoring and setting the numerator equal to zero gives $x = 5/3$. This is not a root of the denominator; therefore, it is a valid x–intercept. □

Problem 20. Use the quadratic formula to solve the equation $x^2 - 6x + 2 = 0$. Simplify as much as possible.

Solution: The solutions are $3 \pm \sqrt{7}$. □

Problem 21. Solve the following equation:

$$\log_2(x+2) + \log_2(x+6) = 5.$$

Solution: Use the properties of logarithms to simplify the right hand side of the equation into a single logarithm; then, convert to exponential form to get $x^2 + 8x + 12 = 32$. Solving this quadratic equation gives $x = 2$ and $x = -10$. But only $x = 2$ can be a solution of the original equation. □

Problem 22. Simplify the following expression

$$\frac{\ln\left(e^{x+2}\right) - \log_7 1}{\log_4 256}.$$

Solution: Using properties of logarithms the previous expression simplifies to $(x+2)/4$. □

Problem 23. Solve the equation $8\ln\left(\frac{1}{2}x\right) = 72$.

Solution: Isolate the natural logarithm, convert to exponential form and solve to obtain $x = 2e^9$. □

Problem 24. Does the following table represent an exponential function? If so, write a corresponding expression:

x	-2	-1	0	1	2	3	4
$f(x)$	0.0125	0.05	0.2	0.8	3.2	12.8	51.2

Solution: The ratio of the $f(x)$–values is 4. Since $f(0) = 0.2$, we can then write $f(x) = 0.2 \cdot 4^x$ □

Problem 25. A real estate developer opens a new subdivision with a certain number of houses. Each year, 12 new houses are built in the subdivision. The number of houses in the subdivision is a linear function of the years since the subdivision opened. Seven years after opening, there were 109 houses. Eleven years after opening, there were 157 houses.

How many houses were there at the beginning? What are the units of the slope of the line?

Solution: Assume t is the number of years since the subdivision opened, and y i the number of houses in the subdivision. The function $y = f(t)$ is linear, and the points $(7, 109)$ and $(11, 157)$ belong in its graph. This gives a slope of $(157 - 109)/(11 - 7) = 12$ houses/year. An equation of the corresponding line is $y - 109 = 12(t - 7)$, or $y = 12t + 25$. There were 25 houses initially. □
