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# Remark on the Isospin Mass Differences

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The isospin splittings of the $\frac{1}{2}^+$ baryon octet are observed empirically to depend upon the (average) sum of charge and hypercharge or equivalently, upon the (average) $V_3 = -\frac{1}{2}(Q+Y)$ spin component. We construct a simple quark model compatible with this observation and then use this model to predict the $\frac{3}{2}^+$ decuplet isospin splittings. One obtains different decuplet predictions from the standard approach, enabling one eventually to test whether or not the baryon regularity is accidental. If nonaccidental, then support is given to recent conjectures that isospin splittings may be partially nonelectromagnetic.

Since the advent of unitary symmetry,[1] the mass splittings between isospin multiplets within an $SU_3$ multiplet have been understood as being due to a tensor which transforms as the hypercharge $Y$. Thus, ignoring electromagnetic and weak interactions, one can write an effective-mass Hamiltonian as

$$H_{\text{strong}} = \lambda_0 \text{Tr}\overline{B}B + \lambda_1 \text{Tr}\overline{B}YB + \lambda_2 \text{Tr}\overline{B}BY + \lambda_3 \text{Tr}\overline{B}YBY\,,$$

where $B$ is the standard $3\times 3$ baryon matrix and where the relatively small 27-plet contribution is usually set equal to zero. The splittings between members of an isospin multiplet have been thought to be purely electromagnetic in origin and thus due to a tensor which transforms as the charge $Q$. Thus one writes the isospin contribution as[2]

$$H_{\text{isospin}} = \delta_1' \text{Tr}\overline{B}QB + \delta_2' \text{Tr}\overline{B}BQ + \delta_3' \text{Tr}\overline{B}QBQ\,,$$

where it is known that the 27-plet contribution cannot be ignored here. The very accurate Coleman-Glashow mass relation,

$$m_{\Xi^-}-m_{\Xi^0}+m_n-m_p=m_{\Sigma^-}-m_{\Sigma^+}$$

for the baryon octet, follows from this assumption. The success of this Hamiltonian for the isospin splittings and the general feeling that they are entirely electromagnetic in nature have motivated a number of attempts to calculate these splittings from a less phenomenological point of view. However, the relationship between the electromagnetic self-mass and the electromagnetic form factors derived by Feynman and Speisman[3] and by Cini, Ferrari, and Gatto[4] were known to be inadequate for this purpose. This led Coleman and Schnitzer[5] and Cottingham[6] to consider additional contributions to the self-mass from other diagrams, additional intermediate states, and corrections for $\phi$-$\omega$ mixing. These and subsequent calculations have met with limited success.[7]

On the other hand, the possibility has always been envisaged that a more general form of isospin splitting could occur[8,9] in which a portion of the contribution was nonelectromagnetic in origin. Recently, there have been conjectures by Pati[10] and by Cabibbo and Maiani[11] that the isospin splitting may contain a nonelectromagnetic contribution. This possibility is also suggested both by the normal decay rate[12] of $\eta\to 3\pi$, which should

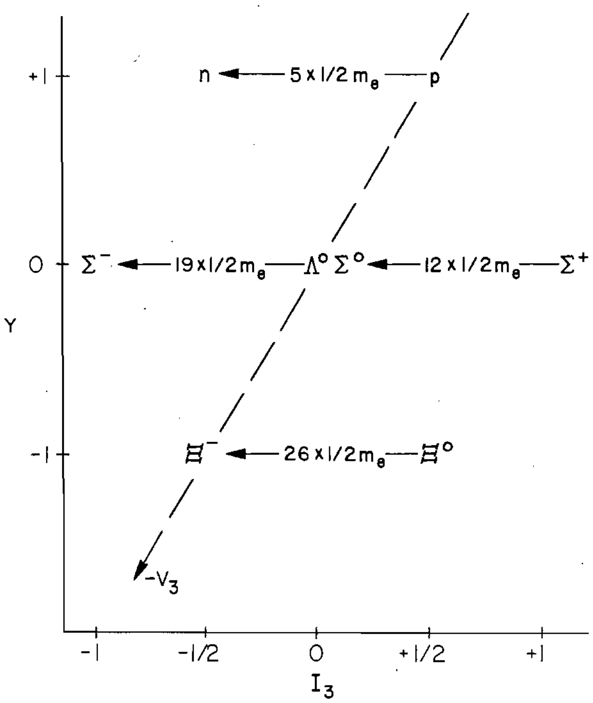


FIG. 1. Approximate isospin splittings of the baryon mass in units of one-half the electron mass.

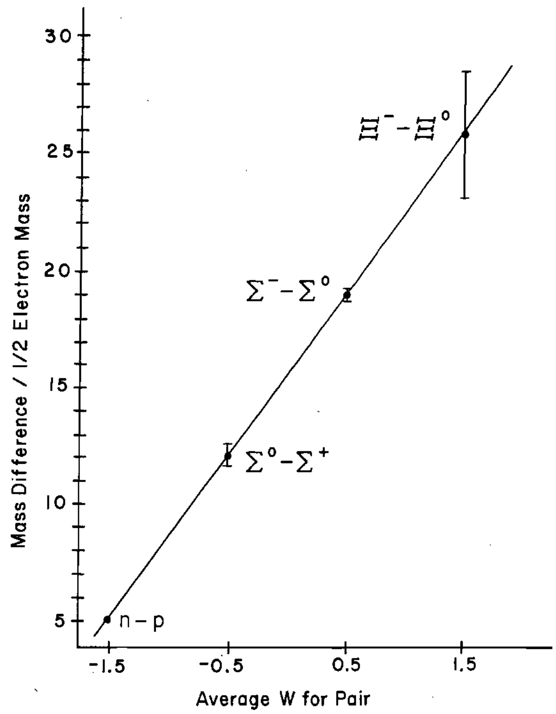


FIG. 2. The linearity of the four isospin splittings. Experimental points and error bars are those of Table I.

theoretically vanish, and by the

$$m_{K^+}{}^2-m_{K^0}{}^2=m_{\pi^+}{}^2-m_{\pi^0}{}^2$$

electromagnetic prediction.[13] In support of the possibility of a nonelectromagnetic contribution, we wish to point out a simple regularity in the baryon octet isospin splittings which suggests that the direction of these splittings is determined by the operator $V=\frac{1}{2}(Q+Y)$ instead of the operator $Q$.

In Fig. 1, we have indicated the approximate isospin splittings of the baryon octet in units of $\frac{1}{2}$ the electron mass. One immediately notices the linear increase of the four splittings as one proceeds in the general direction from $p$ to $\Xi^-$. The accuracy of this linearity is expressed in Fig. 2, using the present experimental points and error bars (see Table I). The straight line was plotted through the approximate values[14] of Fig. 1. As this linearity is so well satisfied, one is tempted to ask if there is some variable in terms of which it can be expressed. Recalling that the axis $V_3$ in the octet weight diagram is defined by a line joining the $p$, $\Sigma^0$, and $\Xi^-$, one observes that the mass splittings become linearly dependent upon the $V_3$ component of a particle. More precisely, $\Delta m$ is linearly dependent upon $\langle V_3\rangle$, the average $V_3$ of the pair, and we may write

$$\Delta m/\Delta V_3=a_1\langle V_3\rangle+a_2\,.$$

TABLE I. The more accurately known electromagnetic splittings.

| | Experimental mass (MeV) | Experimental mass Electron mass = 2.0 |
|---|---|---|
| $m_n - m_p$ | $1.2933 \pm 0.0001$ | $5.0618 \pm 0.0004$ |
| $m_{\Sigma^0} - m_{\Sigma^+}$ | $3.06 \pm 0.13$ | $11.98 \pm 0.51$ |
| $m_{\Sigma^-} - m_{\Sigma^0}$ | $4.86 \pm 0.07$ | $19.02 \pm 0.27$ |
| $m_{\Xi^-} - m_{\Xi^0}$ | $6.6 \pm 0.7$ | $25.83 \pm 2.7$ |
| $m_{K^0} - m_{K^\pm}$ | $3.94 \pm 0.13$ | $15.42 \pm 0.51$ |
| $m_{\pi^\pm} - m_{\pi^0}$ | $4.6041 \pm 0.0037$ | $18.020 \pm 0.014$ |
| $m_e$ | 0.511006 | 2.00 |

It follows from this equation that we can write the electromagnetic portion of the Hamiltonian as

$$H_{\text{isospin}} = \tfrac{1}{2} a_1' V_3^{\,2} + a_2' V_3 + a_3' ,$$

where $a_1'$ and $a_2'$ are constants to be fit to the data. $a_3'$ is undetermined by the splittings and may be ignored or absorbed into the rest of the Hamiltonian. In order that the equation also be valid for the antiparticles, $a_2$ and $a_2'$ must change sign under charge conjugation. This can be achieved by letting $a_2$ and $a_2'$ be proportional to baryon number (or less satisfactorily, by simply using the magnitude of $V_3$ in place of $V_3$). One easily checks that $H_{\text{em}}$ gives two baryon splittings when $a_1$ and $a_2$ are fitted to the other two differences. It may equivalently be stated that we have one new mass relation, e.g.,

$$(m_{\Xi^-} - m_{\Xi^0}) - (m_n - m_p) = 3[(m_{\Sigma^-} - m_{\Sigma^0}) - (m_{\Sigma^0} - m_{\Sigma^+})] .$$

Thus, one could say that the direction of the symmetry breaking appears to be rotated from the expected direction $Q$, through an angle of 30°, to the direction $Q+Y$. One may thus write the isospin portion of the Hamiltonian by replacing the operator $Q$ by the operator $V_3$, giving

$$H_{\text{isospin}} = \delta_1 \mathrm{Tr}\overline{B} V_3 B + \delta_2 \mathrm{Tr}\overline{B} B V_3 + \delta_3 \mathrm{Tr}\overline{B} V_3 B V_3 .$$

The Coleman-Glashow formula is still satisfied and the linearity of the splittings gives $\delta_3 = -2(\delta_1 + \delta_2)$, resulting in two independent parameters $\delta_1$ and $\delta_2$ which are easily shown to satisfy

$$m_n - m_p = -(\delta_1 + \tfrac{1}{2}\delta_2)$$

and

$$(m_{\Sigma^0} - m_{\Sigma^+}) - (m_n - m_p) = \tfrac{1}{2}(\delta_1 + \delta_2) .$$

One can frame the same idea in terms of a quark model. Ordinarily,[15] one writes the isospin contribution as a linear combination of (a) a pairwise quark charge interaction, (b) a sum of the masses of the constituent quarks, and (c) a magnetic moment interaction term. Although the quarks have fractional charge and hypercharge, the combination $(Q+Y) = 2V_3$ is integral having the values $-1$, 0, and $+1$ for the $\mathcal{P}$, $\mathfrak{N}$, and $\lambda$ quarks, respectively. Defining $W \equiv Q + Y$, we may use this generalized type of charge $W$ to carry the $V_3$ dependence. The effective-mass Hamiltonian may be written as

$$H'_{\text{isospin}} = g \sum_{i \neq j} W_i W_j + \sum_i M_i ,$$

representing first, a pairwise Coulomb type of interaction of quarks in terms of the generalized charge and second, a sum over the masses of the constituent quarks. The baryon splittings may then be represented in terms of two constants, $g$ and the mass difference of the neutron and proton quark.[16] A magnetic-moment interaction term is not needed.

Although one can fit the pseudoscalar mesons to this model, the two splittings serve only to fix the two adjustable parameters (which incidentally vary widely from the baryon parameters) and no check can be made. However, one can predict all the decimet splittings in terms of two parameters as we have shown in Table II. These predictions differ from the usual predictions in the following respects. When the symmetry breaking is assumed to be in the $Q$ direction, one finds that

$$m_{\Delta^0} - m_{\Delta^+} = m_{\Sigma^{0*}} - m_{\Sigma^{+*}}$$

and

$$m_{\Delta^-} - m_{\Delta^0} = m_{\Sigma^{-*}} - m_{\Sigma^{0*}} = m_{\Xi^{-*}} - m_{\Xi^{0*}} .$$

On the other hand, if the symmetry-breaking direction is $V_3$, then one finds that none of these equalities holds, but rather that the sequence of splittings should increase linearly as shown in Table II, with the only equality being

$$m_{\Sigma^{0*}} - m_{\Sigma^{+*}} = m_{\Delta^-} - m_{\Delta^0} .$$

The equation

$$m_{\Delta^-} - m_{\Delta^{++}} = 3(m_{\Delta^0} - m_{\Delta^+})$$

holds for either direction of the splitting, as does the Coleman-Glashow formula. The present deci-

TABLE II. Model predictions for the decuplet electromagnetic splittings in terms of parameters $\delta = (m_{\mathfrak{N}} - m_{\mathcal{P}})$ and $g$.

| | Model |
|---|---|
| $m_{\Delta^+} - m_{\Delta^{++}}$ | $\delta$ |
| $m_{\Delta^0} - m_{\Delta^+}$ | $g + \delta$ |
| $m_{\Delta^-} - m_{\Delta^0}$ | $2g + \delta$ |
| $m_{\Sigma^{0*}} - m_{\Sigma^{+*}}$ | $2g + \delta$ |
| $m_{\Sigma^{-*}} - m_{\Sigma^{0*}}$ | $3g + \delta$ |
| $m_{\Xi^{-*}} - m_{\Xi^{0*}}$ | $4g + \delta$ |

met data are not of sufficient accuracy to test the direction of the decimet isospin splittings. However, when these data become available, it will be interesting to see if the baryon-splitting direction holds for other multiplets. Because of the similarity of quark structure of the octet and decimet, the same parameters may fit both multiplets as has been suggested by several authors.

Returning to the pseudoscalar mesons, the presence of a linear term in $V_3$ is not very satisfactory when the baryon number is zero, as the formula seems somewhat contrived. It would have been hoped that a single term quadratic in the splitting operator would be sufficient and consequently one would get one mass relation. Since the baryon splitting takes a simple direction, one can ask if a splitting of the form

$$H_{\text{mesons}} = A(Q + \alpha Y)^2$$

could account for the meson splittings. Here the constant $\alpha$ determines the amount of nonelectromagnetic contribution or, equivalently, the direction of the splitting. One easily determines that

$$\alpha = -\tfrac{1}{2}[(m_{K^0} - m_{K^+})/(m_{\pi^+} - m_{\pi^-}) + 1] = -0.93 .$$

Thus, the splitting direction is rather close (about 2°) to the axis $Q$-$Y$, which is the hypercharge-type axis perpendicular to the $V_3$ axis. A choice of the splitting axis $Q$-$Y$ would imply

$$m_{K^0} - m_{K^\pm} = m_{\pi^\pm} - m_{\pi^0} ,$$

which is rather poor (15%) but would give the correct direction to the splittings.

In conclusion, it would seem useful to define an angle-dependent operator for the mass splittings as

$$Z(\theta) \equiv \cos\theta\, Y + \sin\theta\, Q ,$$

where $\theta = 0$ gives the operator $Y$ for the splittings between isospin multiplets, and $\theta = \pm 90°$ gives the operator $Q$ for purely electromagnetic splittings. Then, intermediate values of $\theta$ for isospin splittings are a measure of the nonelectromagnetic contribution. Our observation is that the baryon octet splittings occur in the direction $\theta = +45°$. Also, the meson splittings appeared to be close to the direction $\theta = -45°$. One could equivalently use the operator

$$Z(\phi) = \cos\phi\, I_3 + \sin\phi\, V_3 ,$$

giving different mixtures of isospin and $V$-spin components and achieving a formal analogy with the mixing of weak currents $I^\pm$ and $V^\pm$.

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[14]One also notices that using a mass unit where the electron-neutrino splitting is 2 results in hadron splittings which are rather close to integral values. This suggests that perhaps some rather simple relations might connect the lepton and hadron isospin splittings. These integral ratios of splittings can also be expressed in another way using the very accurate relation $(m_\mu - m_e)/m_e \approx 3/2\alpha$ pointed out by H. Primakoff in *Proceedings of the International School of Physics Enrico Fermi, Course No. XXXII* (Academic, New York, 1966), p. 97. His relation may be restated as $m_e - m_\nu = \frac{2}{3}\alpha(m_\mu - m_e)$. Thus, if the mass unit $m_\mu - m_e$ is used, then the isospin splittings for leptons and hadrons are close to integral multiples of $\frac{1}{3}\alpha$. That a mass unit of about $m_\pi$ gives isospin splittings of the order of $\alpha$ has been pointed out by many authors since Y. Nambu [Progr. Theoret. Phys. (Kyoto) 7, 595 (1952)].

[15]For example, see B. T. Feld, *Models of Elementary Particles* (Blaisdell, Waltham, Mass., 1969).

[16]As a point of curiosity one can ask what quark isospin splitting is obtained. By shifting the zero of the generalized charge by integral amounts, $W' = W + n$, one obtains a discrete set of values without altering the predictions. For a shift of $n = 1$, one obtains $m_{\mathcal{P}} - m_{\mathcal{N}} = +m_e$. Equivalently, one observes that the electron-neutrino splitting is the next value on the line in Fig. 2 below $n$-$p$. It is interesting that the lepton and baryon isospin splittings lie on the same line.